THREE PHOTON W-STATE

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Multiphoton entanglement is the basis of many quantum communication schemes, quantum cryptographic protocols, and fundamental tests of quantum theory. For entangled three qubit states it has been shown that there are two inequivalent classes of states, under stochastic local operations and classical communications. The classes are represented by the GHZ- and W-state. The GHZ-state has been used to proof Bell's theorem without inequality. Contrary to the GHZ-state, the W-state shows high robustness of entanglement against photon loss. Here we show the first experimental results on the observation of the polarization entangled three-photon W-state from spontaneous parametric down-conversion.

I. INTRODUCTION

Entangled states are key elements in the field of quantum information processing. The experimental preparation, manipulation and detection of multi-photon entangled states is of great interest for implementations of quantum communication schemes, quantum cryptographic protocols, and for fundamental tests of quantum theory. Parametric down conversion has been proven to be the best source of entangled photon pairs in experiments on the foundations of quantum mechanics [1] and in quantum communication. Experimental realizations of concepts like entanglement based quantum cryptography [2], quantum teleportation [3] demonstrated the usability of this source. Greenberger-Horne-Zeilinger (GHZ) have discovered that the entanglement in the three qubit state

$$|GHZ\rangle_{abc} = \frac{1}{\sqrt{2}}(|HHH\rangle + |VVV\rangle)_{abc} \tag{1}$$

leads to a conflict between local realism and nonstatistical predictions of quantum mechanics [4]. Experimentally, the interference of photons generated by independent down conversion processes enabled the first demonstration of a three-photon GHZ-argument [5].

Another three qubit state, the W-state, has recently attracted considerable attention [7],

$$|W\rangle_{abc} = \frac{1}{\sqrt{3}}(|HHV\rangle + |HVH\rangle + |VHH\rangle)_{abc}. \tag{2}$$

It was shown that this state is inequivalent to the GHZ-state under stochastic local measurements and classical exchange of messages [6]. The entanglement in the W state is highly robust against the loss of one qubit, while the GHZ-state will be reduced to a product of two qubits. Also it has been shown recently that the W state allows for a generalized GHZ-like argument against the Einstein-Podolsky-Rosen idea of elements of reality [8]. In this work we shall address two issues: a feasible experimental scheme to produce such states will be presented and we will derive observable properties of the state, together with the Bell-type correlation function associated with it. We will also analyze the violation of local realism by such states. The analysis will be in two steps. First we show to what extent the correlation function for the W state violates the generalized Bell inequality for 3-qubits [9–11]. In the next step we apply the numerical approach of [12] to test the full set of sufficient and necessary conditions for local realism which applies to all predictions for local von Neumann measurements on the state. We also will discuss the robustness of entanglement against photon loss. We will present the first experimental results on the observation of the entangled 3-photon W-state. In conclusion, we will address possible applications for multiparty quantum communication.

II. ENTANGLED THREE-PHOTON W STATE FROM DOWN-CONVERSION

The process of spontaneous parametric down-conversion (SPDC) offers currently the best method for generating entangled photon pairs [13]. In type-II SPDC multiple emission events during a single pump pulse lead to the following state:

$$\exp\left(-i\alpha(\hat{a_0}^\dagger_H \hat{b_0}^\dagger_V + \hat{a_0}^\dagger_V \hat{b_0}^\dagger_H)\right) |0\rangle \,, \tag{3}$$

where C is a normalization constant, α is proportional to the pulse amplitude, and $\hat{a_0}^\dagger_H$ is the creation operator of a photon with horizontal polarization in mode a_0, etc.

The second order term corresponds to the emission of 4 photons and it is proportional to

$$(\hat{a}^\dagger_H \hat{b}^\dagger_V + \hat{a}^\dagger_V \hat{b}^\dagger_H)^2 \,|0\rangle \,. \tag{4}$$

This term can be written as a superposition of photon number states,

$$|\,2H_{a_0}, 2V_{b_0}\rangle + |\,2V_{a_0}, 2H_{b_0}\rangle + |\,1H_{a_0}, 1V_{a_0}, 1H_{b_0}, 1V_{b_0}\rangle \,, \tag{5}$$

where e.g. $2H_{a_0}$ means 2 horizontally polarized photons in the beam (a_0) and $2V_{b_0}$ means 2 vertically polarized photons in the beam (b_0) [9]. Our W state preparation setup is similar to the GHZ setup [5], the main idea is to transform the four photon state into three entangled photons, and the fourth photon is used as trigger. The experimental setup is shown in fig. 1. The photons in arm (a_0) are split by a polarization beam splitter (PBS), such that the V photon is reflected and triggers detector D_T. The photons in arm (b_0) are split by an adjusting beam splitter ($adj.BS$) with a reflection coefficient $R_V = 2R_H$, one output arm continues towards an analyzer D_c, and the second output arm continues towards to a beam splitter BS_1 for an overlap with the photon transmitted from the first PBS. Finally, the two outputs are directed via beam splitter BS_2 to two analyzers D_a and D_b. The four photons are detected behind filters selecting with a frequency band, which is narrower than that of the pump pulses [14]. This ensures that a detected coincidence between D_a, D_b ,D_c, and D_T is corresponding to four photon state giving in eq. 5 [15].

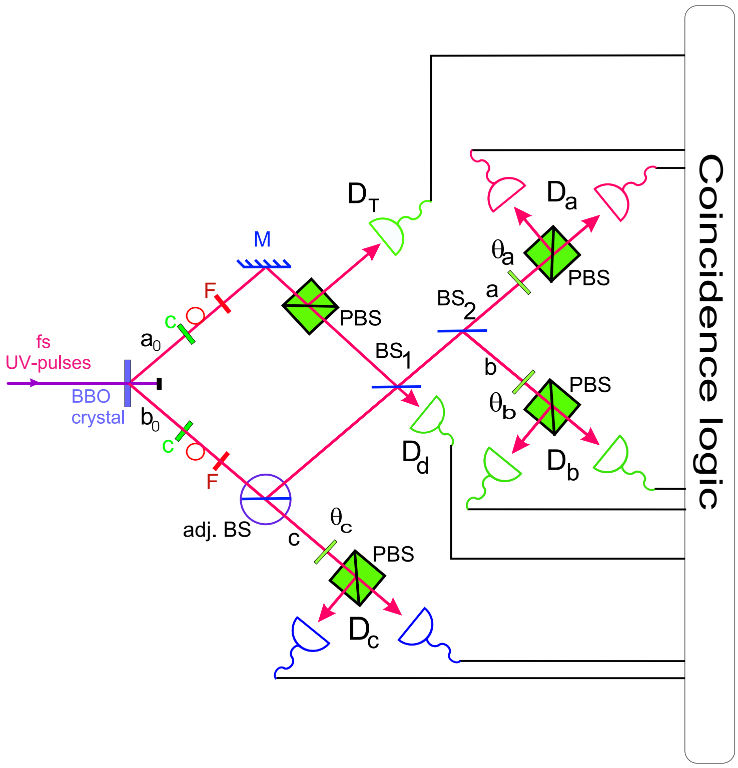

FIG. 1. Experimental setup for the demonstration of entangled three-photon W-state where C, F, M, BS, adj. BS and PBS stand for fiber coupler, filter, mirror, non-polarizing beam splitter, adjusting beam splitter with a reflection coefficient $R_V = 2R_H$ and polarizing beam splitter respectively. Three polarization analyzers with different settings ($\theta_i (i = a, b, c)$) are used.

III. STATE $|W\rangle$ VERSUS LOCAL REALISM

In this section, we address the issue of the violation of local realism of W-state (2). We start deriving the correlation functions associated with it.

A. Quantum correlations

We consider dichotomic local observables with eigenstates given by

$$|k_n, \phi_n, \theta_n\rangle = \cos(\theta_n + k_n \frac{\pi}{4})|V\rangle_n + e^{i\phi_n} \sin(\theta_n + k_n \frac{\pi}{4})|H\rangle_n, \tag{6}$$

where $k_n = \pm 1$ denotes the local results and $n = a, b, c$ enumerates the qubits. The correlation function for three qubit systems is defined as the mean value of the product of the three local results:

$$E = \sum_{k_a, k_b, k_c = \pm 1} k_a k_b k_c \; P(k_a, k_b, k_c) \tag{7}$$

After using simple trigonometric relations, one obtains the following form for the correlation function:

$$\begin{aligned} E = -\frac{2}{3}\Big[&\cos(\phi_b - \phi_c) \cos\tilde{\theta}_a \sin\tilde{\theta}_b \sin\tilde{\theta}_c \\ &+ \cos(\phi_a - \phi_c) \sin\tilde{\theta}_a \cos\tilde{\theta}_b \sin\tilde{\theta}_c \\ &+ \cos(\phi_a - \phi_b) \sin\tilde{\theta}_a \sin\tilde{\theta}_b \cos\tilde{\theta}_c \Big] \\ &+ \cos\tilde{\theta}_a \cos\tilde{\theta}_b \cos\tilde{\theta}_c, \end{aligned} \tag{8}$$

where $\tilde{\theta}_i = 2\theta_i + \frac{\pi}{2}$. For the fixed settings $\phi_a = \phi_b = \phi_c = 0$ and $\theta_b = \theta_c = 0$, the correlation function is simply $\cos(\theta_a)$ which shows perfect correlation (i.e., $|E| = 1$). The correlation function for the W-state therefore has a completely different structure than the one for GHZ-state $E(GHZ) = \cos(\phi_a + \phi_b + \phi_c)$. The correlation function can be also expressed in the following way

$$E = \hat{T} \circ \hat{r}_a \otimes \hat{r}_b \otimes \hat{r}_c, \tag{9}$$

where $\hat{T}$ denotes a *correlation tensor*, $\vec{r}_i = (\cos\tilde{\phi}_i \sin\tilde{\theta}_i, \sin\tilde{\phi}_i \sin\tilde{\theta}_i, \cos\tilde{\theta}_i)$ and $\circ$ denotes the scalar product in R^{27}. The nonvanishing components of the correlation tensor are: $T_{caa} = T_{cbb} = T_{aca} = T_{bcb} = T_{aac} = T_{bbc} = -\frac{2}{3}$, and $T_{ccc} = 1$.

B. Violation of local realism

As it was shown in [9], there exists a single generalized Bell-type inequality which gives the sufficient and necessary condition for the 3-qubit correlation function to have a local realistic model, namely

$$\frac{1}{2^3} \sum_{s_3=\pm1} \sum_{s_2=\pm1} \sum_{s_1=\pm1} | \sum_{m_3=\pm1}^{2} \sum_{m_2=\pm1}^{2} \sum_{m_1=\pm1}^{2} s_1^{m_1} s_2^{m_2} s_3^{m_3} E(m_1, m_2, m_3)| \leq 1, \tag{10}$$

where $m_i = 1, 2$ labels the experiment performed by the i-th local observer. Please note that the inequality applies to experiments in which each local observer can choose between two local observables.

As it was shown in [11] the inequality is satisfied if and only if

$$\max_{\alpha,\beta,\gamma} \Big\{ \sum_{i,j,k=1}^{2} |T_{ijk}| \sin(\alpha + i\frac{\pi}{2}) \sin(\beta + j\frac{\pi}{2}) \sin(\gamma + k\frac{\pi}{2}) \Big\} \leq 1, \tag{11}$$

where T_{ijk} is the correlation tensor in any set of local Cartesian coordinates systems and α, β, γ are arbitrary.

We have solved this optimization using the amoeba numerical procedure. The inequality is maximally violated by the factor of 1.523, which is equivalent to the critical visibility 0.6565. This fully agrees with the result of Cabello [8] which has been obtained via an entirely different approach. We note that Cabello has given a GHZ-proof of Bell 's theorem without inequality specific for the W state [8]. It is also shown that the correlations between two qubits selected from a trio prepared in a W state violate the Clauser-Horne-Shimony-Holt inequality beyond Cirel'son's bound. Such a violation is smaller than the one achieved by two qubits selected from a trio in a GHZ state. However, it has the advantage that all local observers can know from their own measurements whether their qubits belong to the selected pair or not [16].

A linear optimization procedure given by [12] which tests the possibility of quantum probabilities [1] to be described by local realistic models was also applied to the predictions for the W state. The results differ a bit from those obtained using the generalized Bell inequality. They are equivalent to violation of such an inequality by factor 1.5523 ($V'_{crit} = 0.6442$). This is again an interesting feature of the W states, because for GHZ states both the linear optimization procedure of [12] and the one based on inequality (10) give the same result.

IV. ENTANGLEMENT ROBUSTNESS

Entangled states are subject to decoherence and particle losses due to their interaction with the environment. For the three photon W-state, we have analyzed the case when one photon is lost from the mode a. Mathematically this corresponds to trace out one qubit and the density matrix of the remaining qubits becomes

$$\rho_{bc} = Tr_a[\rho^W_{abc}] = \frac{2}{3}\,|\,\phi^+\rangle_{bc}\,\langle\,\phi^+|+\frac{1}{3}\,|\,HH\rangle_{cd}\,\langle\,HH|\,, \tag{12}$$

where the three photons density matrix is defined by $\rho^W_{abc} = |\,W\rangle\,\langle\,W|_{abc}$. A strong criterion for entanglement analysis is the Peres-Horodecki criterion, which states that for separable states ρ, the partial transpose of the density matrix ρ must have nonnegative eigenvalues [18]. We apply this criterion to the above reduced density matrices and obtain $\lambda_k = \{1/3, 1/3, 1/6(1-\sqrt{5}), 1/6(1+\sqrt{5})\}$. The state of Eq.(2) is partially entangled and there are purification procedures to transform this state to a pure entangled state [19]. One can also obtain a state close to a maximally entangled state by means of a filtering measurement [20].

V. EXPERIMENT

In the experiment, we used UV-pulses of a frequency doubled mode-locked Ti:saphire laser with repetition rate of 82 MHz to pump a 2mm thick BBO crystal. The degenerate down-conversion emission at the two characteristic type II crossing points was coupled into single mode fibers to exactly define the emission modes and then filtered with narrow-band interference filters ($\Delta\lambda = 3$ nm). To observe a coherent superposition in a setup presented in section II (see Fig. 1), the photons being overlapped need to be indistinguishable in frequency, time of arrival, and spatial mode at BS_1. To show this coherence one only needs to look at the 1st order process in SPDC where two orthogonally polarized photon pairs are created. Rotating the polarization in one arm by 90 degrees enables us to observe a second order interference effect, the so-called Hong-Ou-Mandel dip, if both photons arrive at BS_1 [21]: The photons do not split up because of their bosonic nature. By adding another detector(D_d) in the unused output arm of BS_1, we observe a dip in the coincidence counts (e.g. in detectors D_a and D_d) for zero delay between the photons. Simultaneously one can observe an increase in the coincidence counts between the detectors D_a and D_b (bump). Theoretically, the dip counts drop to zero and the bump counts double for zero delay; Experimentally, we can define the visibility as $V_{dip} = (c_{max} - c_{min})/c_{max}$ and $V_{bump} = (c_{max} - c_{min})/c_{min}$ for the dip and bump respectively where c_{max} and c_{min} are the maximum and minimum twofold coincidence counts respectively.

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[1] That is, it is more general than the criterion for the correlation function.

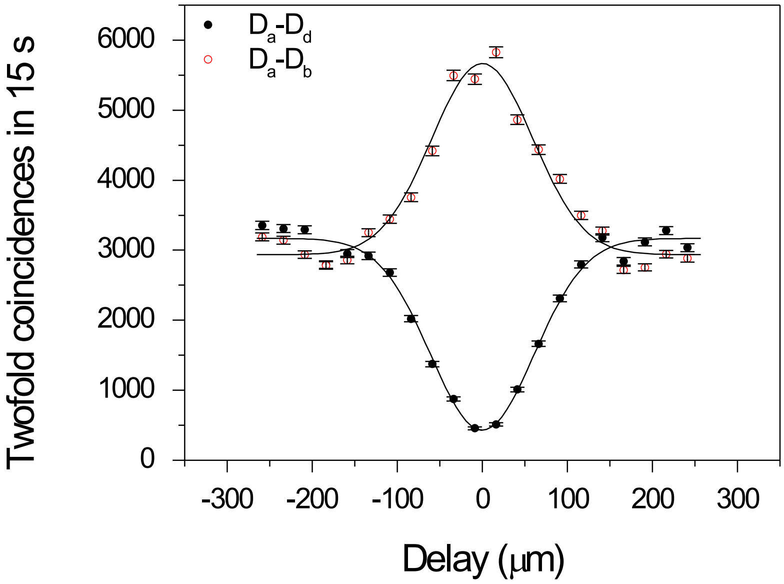

FIG. 2. Experimental data for twofold coincidence counts at detectors D_a and D_d (full circles) and D_a and D_b (open circles) . The maximum interference occurs at zero delay between the photons arriving at BS_1.

Fig 2 shows the twofold coincidence counts at detectors D_a and D_d (full circles) and D_a and D_b (open circles). The maximum interference occurs at zero delay between the photons arriving at BS_1. We obtain a visibility of $V_{dip} = 86.4 \pm 0.4$ and $V_{bump} = 93.3 \pm 0.6$. We use quarter and half wave plates to set the orientation of the analyzers. The four photons were detected by single photon Si avalanche photodiodes and analyzed with an eight-channel multi-coincidence logic. Fig. 3 shows the 8 possible three-fold coincidences when all three polarization analyzers are oriented along Horizontal/Vertical directions. We clearly observe the superposition of the three terms HHV, HVH, and VHH. The errors given are deduced from propagated Poissonian counting statistics of the raw detection events. The experimental analysis of W state along $+45/-45$ linear polarization, and along left/right circular polarization, the study of the correlation function and violation of local realism are in progress [22].

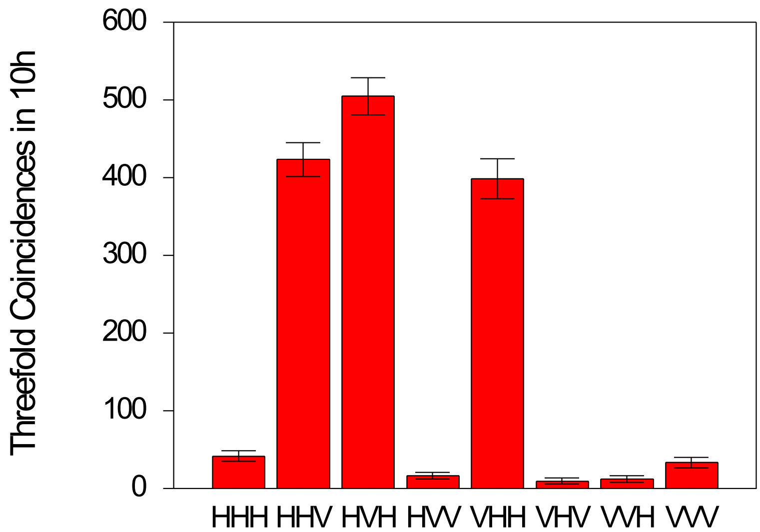

FIG. 3. Threefold photon coincidences in 10 hours of measurement for the entangled 3-photon W state (Eq. 2) with Horizontal/Vertical polarization settings.

VI. CONCLUSIONS

In conclusion, we have presented an experimental setup and have shown first results on the observation of W-state. We have derived a Bell-type quantum correlation function for the W-state, and we have also analyzed the violation of local realism. Perfect correlations together with the violation of a generalized Bell inequality are ingredients for secure multi-party quantum communication [23,24]. As it has been shown very recently, the W state can be used for multi-key distribution and secret sharing protocols. [25].

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